

Thermodynamics of Phase Changes in Systems BaS-Ln₂S₃ (Ln = Pr, Sm, Gd, Tb, Er, Lu)

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Thermodynamic analyses of phase changes were carried out for the systems BaS-Ln₂S₃ with Ln = Pr, Sm, Gd, Tb, Er, and Lu, and the phase diagrams have been experimentally constructed. Formation of the Ba₃Ln₂S₆ sulfides with Ln = Tb through Lu was predicted from the thermodynamic behavior of melts, and the existence of these phases has been experimentally confirmed. Heats of melting were estimated for the sulfides Ln₂S₃, BaLn₂S₄, and Ba₃Ln₂S₆.

1. Experimental Approach

The phase diagrams of the systems BaS-Ln₂S₃ with Ln = Pr, Sm, Gd, Tb, Er, and Lu have been experimentally constructed (Ref 1, 2). The diagrams of BaS-Pr₂S₃, BaS-Sm₂S₃, BaS-Tb₂S₃, and BaS-Lu₂S₃ are shown in Fig. 1 as representative examples. Extended terminal solid solutions of BaS in Ln₂S₃ were observed for the lighter lanthanons in the lanthanide series La through Lu, wherein both atomic numbers and atomic weights monotonically increase from La to Lu but atomic sizes decrease. In the specific case of Ln = Pr, the terminal solution of BaS in Ln₂S₃ extends to approximately equimolar stoichiometry and in the BaS-rich region of the solution, the crystal structure tends to order with a Th₃P₄-type structure. The compound BaLn₂S₄ is formed in the phase diagrams of Ln = Sm, Gd, Tb, Er, and Lu; the compound Ba₃Ln₂S₆ is additionally formed in the phase diagrams of Ln = Tb, Er, and Lu; and a third compound, BaLu₈S₁₃, is formed in the BaS-Lu₂S₃ system.

Experimental determinations of the heats of melting of these sulfide compounds have not been done because measurements are difficult with large heat values and with melting occurring in the temperature range ~1900 K and above. Estimation of these heats of melting can be estimated thermodynamically from the temperature-composition trends of the equilibrium phase boundaries determined for the construction of the phase diagrams. Thermodynamic analysis of an equilibrium solution boundary, in the present case a melt solution boundary, allows one to predict phase formation from the deviation of the solution from ideality. The validity of such predictions can be tested by experiment. For the present purposes, interest is in the contours of the liquidus in equilibrium with one or another of the solid sulfides. The work was undertaken to estimate heats of melting of the solid sulfides from the thermodynamic relationships between such heats and the compositional differences between liquidus and solid phases across equilibrium two-phase regions.

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2. Estimated Heats of Melting (Fusion)

The compounds BaLn₂S₄ that occur in the heavier lanthanon systems have been found to melt congruently and for experimentally determined systems wherein a eutectic reaction occurs between BaLn₂S₄ and Ln₂S₃. Estimates of the heats of melting of these two compounds were made from the compositional difference between liquidus and compound at temperatures between the eutectic temperature, T_{eu} , and compound melting, T_m , and in the composition region between the eutectic, X_{eu} , and the solid phase boundary, X_c . Other regions between compounds were treated analogously. The experimental thermal analysis data for compositions were used to estimate heats of melting with Van Laar's equation in the form

$$\ln (X_i^L/X_i^S) = (\Delta H_m/R)[(1/T_m) - (1/T)] \quad (\text{Eq 1})$$

where R is the gas constant, ΔH_m is the heat of melting of the i th compound of interest, X_i is the mole fraction of the i th compound, the superscripts L and S represent liquid and solid, respectively, T_m is defined above, and T is the temperature at which the X_i^S are evaluated. In the temperature range between T_{eu} and T_m , ΔH_m can be taken as a constant so Eq 1 can be rearranged in linear form to define a relationship defining T^{-1} in terms of $\ln (X_i^L/X_i^S)$ to develop a set of relations from experimental conodes for evaluation of ΔH_m by utilizing a regression method. Results of such estimates for Ln₂S₃ are given in Table 1 and for BaLn₂S₄ in Table 2.

Table 1 Estimates of the heats of melting in simple sulfides

Sulfide	ΔH , kJ/mol
Lu ₂ S ₃	684
Er ₂ S ₃	152
Tb ₂ S ₃	94
Gd ₂ S ₃	60
Sm ₂ S ₃	88
Pr ₂ S ₃	94
BaS	57

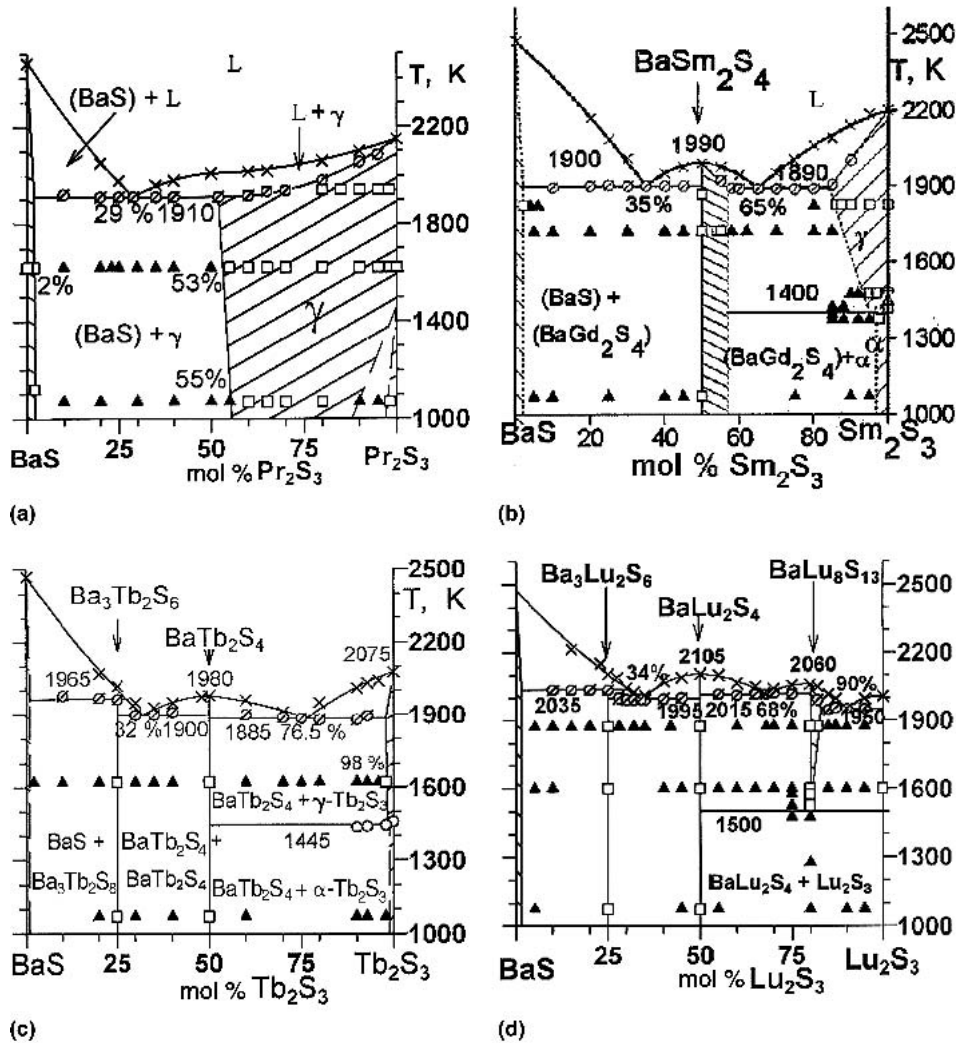


Fig. 1 Representative phase diagrams of BaS- Ln_2S_3 systems for Ln = (a) Pr, (b) Sm, (c) Tb, and (d) Lu illustrating the changing patterns of phase equilibria across the lanthanide series. X-ray and microstructural analyses were used to determine the phase fields with symbols (\blacktriangle) representing two-phase regions and (\square) representing single-phase regions. Thermal analyses were used to determine (\circ) phase transitions, (\emptyset) onset of melting, and (\times) complete melting.

These estimates of ΔH_m s are, as a whole, in agreement with other information (Ref 3, 4). Nevertheless it is necessary to note the obviously high values from the estimates for simple Ln_2S_3 sulfides for Ln = Er and Lu at heavy end of the lanthanide series. These high values are believed to be caused by experimental difficulties that limit accurate determination of the compositional liquidus-solidus ratio at the Ln_2S_3 -rich end of the phase diagrams. Nevertheless there is without doubt a considerable decrease in ΔH_m for each sulfide in the progression from Lu downward toward La. This correlates with a decrease in hardness of Ln^{3+} when viewed as Pearson's acid particles (Ref 5) and reflects the degree of filling of 4f electronic states. It may be noted that the ranges of homogeneity of the intermediate compounds are quite limited and that the decrease in values of ΔH_m from system to system in Tables 1 and 2 are consistent with the trends in values of $T_m - T_{eu}$ and $X_{eu} - X_{comp}$ with

Table 2 Estimates of the heats of melting in binary sulfides

Sulfide	ΔH (BaLn_2S_4), kJ/mol
BaLu_2S_4	307
BaEr_2S_4	264
BaTb_2S_4	358
BaGd_2S_4	94
BaSm_2S_4	84

regard to the position of a lanthanon in the series. These differences combine to produce the observed trend of decrease in ΔH_m through the progression from Lu toward La. If ΔH_m of BaTb_2S_4 is regarded as an anomaly and the ΔH_m values in Table 2 are plotted against atomic number or against the radius of the lanthanide ion $r_{\text{Ln}^{3+}}$, ΔH_m drops

below zero before Pr is reached, and it may be observed that, in the BaS-Pr₂S₃ phase diagram, BaPr₂S₄ does not occur.

3. Excess Partial Molar Gibbs Energies

A thermodynamic estimation of the deviation from ideality of the component compounds forming BaS-Ln₂S₃ liquids were done by combining experimental data from thermal analyses with the Van Laar equation in the form

$$\bar{G}_i^E = (\Delta H_m/T_m)(T - T_m) + RT \ln (X_i^S/X_i^L) \quad (\text{Eq 2})$$

where \bar{G}_i^E is the excess partial molar Gibbs energy of the component of interest in the melt, with X_i^S and X_i^L defined as before with i being the compound of interest. Except for the Pr system, data for X_i^S for the solidus end of a conode

were obtained by fitting the limited number of experimental datum points to second-degree polynomials by the method of least squares. This allows interpolation to evaluate X_i^S values at desired temperature points to define conodes for the available experimental liquidus points. Fitting was done with a complex program, Phase Gramer, which also evaluated all thermodynamic values of interest.

Results from this estimating procedure are shown in Table 3 for the BaS-Pr₂S₃ system; note that the scale for the compositions of BaS invert the scales of Fig. 1 with X_{BaS} for pure BaS being 1 rather than 0. From this table it is evident that for BaS the melt is characterized by a negative deviation from ideality. For the eutectic, the data indicate an excess integral Gibbs energy of $G^E = -5.9$ kJ/mol and produce a Hildebrand energy of interchange reaction of $A_G = -28.4$ kJ/mol. The temperature dependence of \bar{G}_{BaS}^E is

Table 3 Excess partial Gibbs energies of BaS and Pr₂S₃ in the BaS-Pr₂S₃ systems

Temperature and compositions of liquidus and solidus in region between BaS and eutectic				Temperature and compositions of liquidus and solidus in region between Pr ₂ S ₃ and eutectic			
T, K	X _{BaS} ^L	X _{BaS} ^S	\bar{G}_{BaS}^E , kJ/mol	T, K	X _{Pr₂S₃} ^L	X _{Pr₂S₃} ^S	\bar{G}_{Pr2S3^E , kJ/mol
				2100	0.90	0.95	-1.2
				2070	0.80	0.91	-1.3
2060	0.80	0.98	-6.0	2060	0.80	0.89	-2.1
2050	0.80	0.98	-6.2				
				2020	0.65	0.85	-1.2
				2015	0.60	0.85	-0.1
				2010	0.60	0.85	-0.3
				2010	0.50	0.83	2.3
1980	0.75	0.98	-6.9	1980	0.40	0.80	4.0
				1960	0.35	0.73	3.7
				1955	0.35	0.73	3.4
				1910	0.29	0.55	-0.3
					0.29	0.54	-0.6
					0.29	0.52	-1.2
					0.29	0.52	-1.2
					0.29	0.52	-1.2
1910	0.71	0.98	-7.8		0.29	0.50	-1.8

Table 4 Excess partial Gibbs energies of the phase components of melts in the systems of BaS-BaLn₂S₄

Temperature and compositions of liquidus and solidus in region between BaS and eutectic				Temperature and compositions of liquidus and solidus in region between BaLn ₂ S ₄ and eutectic			
T, K	X _{BaS} ^L	X _{BaS} ^S	\bar{G}_{BaS}^E , kJ/mol	T, K	X _{BaLn₂S₄} ^L	X _{BaLn₂S₄} ^S	$\bar{G}_{\text{BaLn2S4}^E$, kJ/mol
BaS-BaSm₂S₄				BaS-BaSm₂S₄			
2170	0.60	0.98	1.9	1975	0.90	1.00	1.1
2085	0.50	0.98	2.8	1950	0.80	1.00	1.9
2010	0.40	0.96	4.0	1900 (eut.)	0.70	1.00	1.8
1900 (eut.)	0.30	0.96	5.2				
BaS-BaGd₂S₄				BaS-BaGd₂S₄			
2200	0.70	0.98	-0.1	1990	0.90	1.00	1.3
2100	0.60	0.98	0	1960	0.80	1.00	1.8
2005	0.50	0.96	0.1	1915	0.70	1.00	1.7
1950	0.40	0.96	2.2	1890 (eut.)	0.66	1.00	1.7
1890 (eut.)	0.34	0.96	2.9				

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well described by a linear relationship, $\bar{G}_{\text{BaS}}^{\text{E}} = \bar{H}_{\text{BaS}}^{\text{E}} - T\bar{S}_{\text{BaS}}^{\text{E}}$, with parameters $\bar{H}_{\text{BaS}}^{\text{E}} = 30$ kJ/mol and $\bar{S}_{\text{BaS}}^{\text{E}} = -12$ J/mol · K with a correlation coefficient of $r > 0.99$. From the BaS-Pr₂S₃ diagram in Fig. 1 it is evident that the Pr₂S₃-rich liquidus has two inflections, which reflects in Table 3 as negative deviations from ideality above 50 mol% Pr₂S₃ and below ~30 mol% Pr₂S₃ but with positive deviations in the region between. Such positive deviations indicate that intermediate phases should not form in the 35-50 mol% Pr₂S₃ region of this system and accounts for the absence of BaPr₂S₄ in this system. The reversion to negative deviations at compositions below 30 mol% make likely the existence of associates in the liquid phase with the rapid decrease in the negative interaction pointing to a ratio of 3:1 for the BaS:Pr₂S₃ ratio and is indicative of the possible existence of phases such as Ba₃Ln₂S₆ in the systems with heavier lanthanons.

The estimating procedure was also applied to other BaS-Ln₂S₃ systems. Results for the Sm and Gd systems are shown in Table 4. Excess integral Gibbs energies and Hildebrand energies of exchange are: for the BaS-Sm₂S₃ system $G^{\text{E}} = 2.9$ kJ/mol and $A_{\text{G}} = 13.6$ kJ/mol and for BaS-Gd₂S₃ system $G^{\text{E}} = 1.9$ kJ/mol and $A_{\text{G}} = 8.4$ kJ/mol. In both systems the melts are characterized by positive deviations from ideality. The temperature dependence of $\bar{G}_{\text{BaS}}^{\text{E}}$ is for both systems well described by linear relationships of the

form $\bar{G}_{\text{BaS}}^{\text{E}} = \bar{H}_{\text{BaS}}^{\text{E}} - T\bar{S}_{\text{BaS}}^{\text{E}}$ with parameters $\bar{H}_{\text{BaS}}^{\text{E}} = 29$ kJ/mol and $\bar{S}_{\text{BaS}}^{\text{E}} = -13$ J/mol · K with a correlation coefficient of >0.99 for the BaS-Sm₂S₃ system and $\bar{H}_{\text{BaS}}^{\text{E}} = 21$ kJ/mol and $\bar{S}_{\text{BaS}}^{\text{E}} = 9.8$ J/mol · K with a correlation coefficient of 0.86 for the BaS-Gd₂S₃ system. The decreasing positive deviation from ideality in progressing from Sm to Gd leads to expectation of additional compounds in heavier lanthanons. An estimate of likely stoichiometry can be made by noting that, up to about 75 mol% BaS in the BaS-Gd₂S₃ system, the melt is nearly ideal, but at lower BaS compositions Table 4 shows that there is a sharp increase by an order of magnitude in $\bar{G}_{\text{BaS}}^{\text{E}}$. This means that the preferable composition for the formation of an intermediate phase in the next heavier lanthanon is Ba₃Ln₂S₆ at a BaS:Ln₂S₃ ratio of 3:1. This prognosis is in agreement with the indications in the results from the BaS-Sm₂S₃ system, and such a phase has been found in the BaS-Tb₂S₃ system and in all heavier lanthanon systems.

Table 5 shows excess integral Gibbs energies and Hildebrand energies for the series Ln = Gd, Tb, Er in BaS-Ln₂S₃ alloys. There is a decrease in both the integral molar Gibbs energy and in the Hildebrand energy from one element to the next. This allows the prediction that an additional phase should exist for heavier lanthanons with anticipation that it might occur in the next lanthanon in the series, which is Tm. It has already been shown that such a compound occurs for Lu at a stoichiometry of BaLu₈S₁₃. Again in the BaS-Er₂S₃ system the partial molar Gibbs energy $\bar{G}_{\text{BaEr}_2\text{S}_4}^{\text{E}}$ is well fit by a linear relationship with $\bar{H}_{\text{BaEr}_2\text{S}_4}^{\text{E}} = -120$ kJ/mol and $\bar{S}_{\text{BaEr}_2\text{S}_4}^{\text{E}} = -57$ J/mol · K with a correlation coefficient of 0.99.

Table 5 Excess integral Gibbs energies and interchange energies at eutectic temperatures and compositions in BaLn₂S₄-Ln₂S₃ systems

System	G^{E} , kJ/mol	A_{G} , kJ/mol
BaGd ₂ S ₄ -Gd ₂ S ₃	1.5	6.2
BaTb ₂ S ₄ -Tb ₂ S ₃	-2.2	-8.8
BaEr ₂ S ₄ -Er ₂ S ₃	-4.4	-17.8

*The integral Gibbs energy is readily computed as $G^{\text{E}} = X_{\text{BaS}}\bar{G}_{\text{BaS}}^{\text{E}} + X_{\text{BaLn}_2\text{S}_4}\bar{G}$ is computed from the Hildebrand relations $\bar{G}^{\text{E}} = A_{\text{G}}X_{\text{BaS}}X_{\text{BaLn}_2\text{S}_4}$.

Table 6 Excess partial molar Gibbs energies of components in the BaLu₂S₄-BaLu₈S₁₃ system

Compositions between eutectic and BaLu ₂ S ₄				Compositions between BaLu ₈ S ₁₃ and eutectic			
T , K	$X_{\text{BaLu}_2\text{S}_4}^{\text{L}}$	$X_{\text{BaLu}_2\text{S}_4}^{\text{S}}$	$\bar{G}_{\text{BaLu}_2\text{S}_4}^{\text{E}}$, kJ/mol	T , K	$X_{\text{BaLu}_8\text{S}_{13}}^{\text{L}}$	$X_{\text{BaLu}_8\text{S}_{13}}^{\text{S}}$	$\bar{G}_{\text{BaLu}_8\text{S}_{13}}^{\text{E}}$, kJ/mol
2100	0.83	1.00	2.5	2055	0.83	1.00	2.6
2065	0.67	1.00	1.0	2040	0.67	1.00	4.6
2050	0.50	1.00	3.8	2015 (eut.)	0.60	1.00	3.6
2020	0.40	1.00	3.0				
2015 (eut.)	0.40	1.00	2.2				

Table 7 Excess partial molar Gibbs energies of components in the BaLu₈S₁₃-Lu₂S₃ system

Compositions between eutectic and BaLu ₈ S ₁₃				Compositions between Lu ₂ S ₃ and eutectic			
T , K	$X_{\text{BaLu}_8\text{S}_{13}}^{\text{L}}$	$X_{\text{BaLu}_8\text{S}_{13}}^{\text{S}}$	$\bar{G}_{\text{BaLu}_8\text{S}_{13}}^{\text{E}}$, kJ/mol	T , K	$X_{\text{Lu}_2\text{S}_3}^{\text{L}}$	$X_{\text{Lu}_2\text{S}_3}^{\text{S}}$	$\bar{G}_{\text{Lu}_2\text{S}_3}^{\text{E}}$, kJ/mol
2050	0.90	1.00	0.7	1995	0.75	1.00	1.4
2015	0.75	0.95	-1.0	1975	0.60	1.00	-1.8
1995	0.65	0.90	-1.8	1950 (eut.)	0.50	1.00	-7.5
1950 (eut.)	0.50	0.85	-3.6				

Between BaLu_2S_4 and Lu_2S_3 an intermediate compound $\text{BaLu}_8\text{S}_{13}$ is interposed between the two end constituents, BaLu_2S_4 and Lu_2S_3 . The compound melts congruently and decomposes eutectoidally. It participates in two eutectic reactions, one with BaLu_2S_4 and the other with Lu_2S_3 . Here it is convenient to consider separately the two subsystems BaLu_2S_4 - $\text{BaLu}_8\text{S}_{13}$ and $\text{BaLu}_8\text{S}_{13}$ - Lu_2S_3 . Data for the excess partial Gibbs energies in these two subsystems are shown in Tables 6 and 7. In Table 6, the excess integral Gibbs energy associated with the eutectic reaction is $G^E = -3.0$ kJ/mol and the Hildebrand exchange energy* is $A_G = -12.6$ kJ/mol. In Table 7 the same values for the second eutectic are, respectively, $G^E = -5.6$ kJ/mol and $A_G = -22.6$ kJ/mol. The temperature dependences of $\bar{G}_{\text{BaLu}_8\text{S}_{13}}^E$ and $\bar{G}_{\text{Lu}_2\text{S}_3}^E$ are again well described by linear relationships with correlations of 0.99 in both cases with $\bar{H}_{\text{BaLu}_2\text{S}_4}^E = -87$ kJ/mol and $\bar{S}_{\text{BaLu}_2\text{S}_4}^E = -43$ J/mol K for $\text{BaLu}_8\text{S}_{13}$ and $\bar{H}_{\text{Lu}_2\text{S}_3}^E = -400$ kJ/mol and $\bar{S}_{\text{Lu}_2\text{S}_3}^E = -200$ J/mol K for Lu_2S_3 .

4. Conclusion

Data from thermal analyses from $\text{BaS-Ln}_2\text{S}_3$ specimens of different compositions have been used to calculate useful

thermodynamic quantities for the systems $\text{BaS-Ln}_2\text{S}_3$ with $\text{Ln} = \text{Pr, Sm, Gd, Tb, Er, and Lu}$. Results from these calculations show deviations of melt solutions from ideality and can be interpreted in terms of phase formation and to predict phase formation in other systems. Utility of the approach for use with other materials is implicit.

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